

# Passive Underwater Imaging Through Optimized Planar Arrays of Hydrophones

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**Abstract**—Planar arrays of acoustic sensors represent a critical component for a 3-D wideband passive imaging system. Recently, a method has been proposed for the design of planar microphone arrays that are superdirective (to provide good performance at low frequencies) and aperiodic (to avoid grating lobes at high frequencies). Robustness against dispersion in the sensor characteristics is achieved by exploiting the probability density functions of such errors. Here, such a method is used to demonstrate that 36 low-cost poorly-matched hydrophones are sufficient to populate a square aperture with a side length of 60 cm used as sensor array for an underwater passive system working from 1.5 kHz to 12 kHz.

**Keywords**—Array signal processing; underwater acoustics; planar array design; passive sonar imaging; digital wideband beamforming

## I. INTRODUCTION

Acoustic imaging of underwater sources by means of passive sonar systems can be applied to target surveillance and tracking, observation and quantification of biological organisms, mapping and analysis of noise sources, and localization of divers, as recently highlighted in [1,2]. To be commercially interesting, a passive sonar system for underwater imaging should be small, lightweight and low-cost. From a technical point of view, the system should be able to process whatever received signal, over a very wide bandwidth (e.g., ranging from a few kilohertz to a few tens of kilohertz), and to generate a 3-D map. Matching technical objectives and commercial requirements it is not trivial.

A planar array of hydrophones connected with a 3-D digital beamformer seems the most natural option [2,3]. However, if the frequency interval under consideration is extremely wide, two distinct problems [4] may arise: in the lowest portion of the band, the wavelength is comparable to or larger than the array aperture, dramatically reducing the array's directivity; and in the highest portion of the band, the short wavelengths require a large number of transducers regularly positioned over the array aperture to prevent spatial under-sampling (i.e., grating lobes).

Recently, a design method [5] has been proposed to optimize both the positions of a planar array transducers and the frequency responses of the FIR filters used into a filter-and-sum beamforming connected to the optimized array. The aim of this method is to solve the two above-mentioned problems.

However, the method in [5] is primarily aimed at designing airborne passive acoustic imaging systems (often referred to as acoustic cameras), deploying a planar array of microphones.

This paper investigates the design of a passive sonar system for underwater acoustic imaging by exploiting the method in [5], taking into account the requirements and peculiarities of a planar array of hydrophones. Specifically, we concentrate on the design of an imaging system performing data-independent 3-D beamforming, aimed at spatially processing far-field signals collected via a planar array.

The paper is organized as follows. Section II introduces the superdirectivity and aperiodic layout for planar arrays. Section III summarizes the method proposed in [5] for the layout and filter bank design. Section IV describes the results achieved for the synthesis of an hydrophone array. Finally, some conclusions are drawn in Section V.

## II. WIDEBAND PLANAR ARRAYS

In 3-D digital beamforming systems deploying a planar array of sensors, a beam is created and steered in different directions to investigate a given portion of space. When the spectra of the signals to be processed encompass a wide frequency interval, as usually occurs in passive systems, the adoption of filter-and-sum beamforming is commonly pursued to achieve optimized performance over the entire band of interest [6]. In these cases, both the array and FIR filters should be properly designed to obtain the desired functionality and avoid any excess in complexity or cost. In addition, if the optimized FIR filters keep their validity for whatever steering direction of interest, the steering direction can be tuned acting only on signal delays, keeping unaltered the array layout and FIR filters.

Regarding the two problems mentioned in the previous section, they can be addressed through superdirective beamforming, at low frequencies, and through the use of sparse aperiodic layouts, at high frequencies.

The superdirectivity theory [7] provides solutions that allow an increase in the array directivity with respect to the directivity obtained with uniform weight coefficients. However, the application of this theory in real systems has long been inhibited due to the inadequate robustness of such solutions to array imperfections and the random fluctuation of

the transducer characteristics. Moreover, while the use of superdirectivity in broadband signal processing through filter-and-sum beamforming has become quite common, the application of superdirectivity to planar arrays has rarely been considered.

Aperiodic array layouts can be used to prevent the appearance of grating lobes when the inter-element spacing exceeds half a wavelength [6], thus enabling the design of sparse arrays with a limited number of transducers. The removal of grating lobes is generally accompanied by an enlargement of the main lobe and an increase in the side-lobe level. A variety of designs have been proposed for sparse aperiodic arrays, utilizing analytical, stochastic or hybrid approaches.

Although the design of both the layout and weight coefficients for a planar array has been addressed by a variety of design methods, the processing of broadband signals through a filter-and-sum beamformer, connected to a planar array with a sparse aperiodic layout, has been rarely considered. Moreover, to date, superdirectivity and aperiodic array design have been mainly considered as two distinct topics.

### III. APERIODIC AND SUPERDIRECTIVE DESIGN

The design method proposed in [5] enables the joint design of the planar array layout and the FIR filters to create a 3-D beamforming system for processing broadband far-field signals that are both robustly superdirective and spatially undersampled (at the two extremes of the band, respectively). Such a method exhibits the following characteristics: filter coefficients and transducer positions are optimized in parallel through an iterative hybrid strategy, which is analytical for the coefficients and stochastic for the positions; the beam pattern energy is minimized and the side-lobe level is kept under control; robustness against dispersion in the transducer characteristics is achieved by exploiting the probability density functions (PDFs) of the array errors; the obtained solution retains its validity for whatever steering direction falling inside a predefined 3-D sector.

In this paper, we consider a planar array, placed on the plane  $z = 0$ , composed of  $N$  point-like omnidirectional transducers, each of which is connected to an FIR filter with an  $L$  tap coefficients. To take into account the effect of the array errors, a random complex variable  $A_n$ ,  $A_n = a_n \exp(-j\gamma_n)$ , constant in time and frequency, should be introduced to model the gain  $a_n$  and the phase  $\gamma_n$  of the  $n$ -th transducer's response. Assuming that a plane wave of frequency  $f$ , coming from the direction of arrival indicated by the azimuth angle  $\varphi$  and the elevation angle  $\theta$  [6] impinges with speed  $c$  on the array, the actual array beam pattern (BP) can be written [5, 6] as follows:

$$B(u, v, f) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} w_{n,l} A_n \exp\left\{j2\pi f \left(\frac{x_n u + y_n v}{c} - l T_c\right)\right\}$$

where  $x_n$  and  $y_n$  are the coordinates of the  $n$ -th transducer,  $w_{n,l}$  is a real value representing the  $l$ -th tap coefficient of the  $n$ -th filter and  $T_c$  is the sampling interval. The variables  $u$  and  $v$  are defined as follows:

$$u = \sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0$$

$$v = \sin \theta \sin \varphi - \sin \theta_0 \sin \varphi_0$$

where  $\varphi_0$  and  $\theta_0$  are the azimuth and elevation angles of the steering direction [6], respectively. If all possible steering directions are taken into account, the variables  $u$  and  $v$  should range in the interval  $[-2, 2]$ . However, an enlargement of the domain of  $u$  and  $v$  over which BP is optimized will reduce the overall performance. To limit the enlargement of this domain, only the steering directions that are effectively used should be considered, optimizing over the rectangle  $u \in [u_{\min}, u_{\max}]$ ,  $v \in [v_{\min}, v_{\max}]$ .

The optimization aim is to minimize the BP energy over the considered  $(u, v)$  rectangle and a given frequency interval, as well as to assure that the signal from the steering direction is delayed without attenuation or distortion. To this end, in [5] the following cost function is proposed:

$$J(\mathbf{w}, \mathbf{x}) = \int_{f_{\min}}^{f_{\max}} \int_{u_{\min}}^{u_{\max}} \int_{v_{\min}}^{v_{\max}} H(u, v) |B(u, v, f)|^2 dv du df +$$

$$+ C \int_{f_{\min}}^{f_{\max}} |B(0, 0, f) - e^{-j2\pi f \Delta}|^2 df$$

where  $f_{\min}$  and  $f_{\max}$  are the bounds of the frequency interval,  $\mathbf{w}$  is a column vector whose elements are the  $M$  filter taps,  $w_{n,l}$ ,  $M = NL$ ,  $\mathbf{x}$  is the vector of the transducer positions,  $H(u, v)$  is a non-negative weighting function,  $C$  is a real constant that tunes the trade-off between the minimization of the BP energy and the BP adherence in the steering direction to the above-mentioned constraint, and  $\Delta$  is the time delay associated with the linear phase of BP in the steering direction. Although the value of  $H(u, v)$  can be driven by the specific optimization target, it basically enables the control of the side-lobe level.

The cost function  $J(\mathbf{w}, \mathbf{x})$  depends on a specific realization of the transducer characteristics  $(A_0, \dots, A_{N-1})$ . To render BP robust against such fluctuations and errors, the optimization of the mean performance of the array is adopted [5]. The mean value of the cost function,  $J_e(\mathbf{w}, \mathbf{x})$ , is computed by considering the PDFs of the transducer characteristics, as follows:

$$J_e(\mathbf{w}, \mathbf{x}) = E\{J(\mathbf{w}, \mathbf{x})\} =$$

$$= \int_{A_0} \dots \int_{A_{N-1}} J(\mathbf{w}, \mathbf{x}) f_{A_0}(A_0) \dots f_{A_{N-1}}(A_{N-1}) dA_{N-1} \dots dA_0$$

where  $f_{A_n}(A_n)$  is the PDF of the random variable  $A_n$ .

The minimization of  $J_e(\mathbf{w}, \mathbf{x})$  should be performed with respect to the vectors  $\mathbf{w}$  and  $\mathbf{x}$ . It has been shown in [5] that, if the vector  $\mathbf{x}$  is kept fixed, the resulting function for  $\mathbf{w}$  has only a minimum that can be determined analytically. In contrast, by fixing  $\mathbf{w}$ , the resulting function for  $\mathbf{x}$  typically has many local minima (because the elements of  $\mathbf{x}$  appear in the exponentials of the BP equation), and no analytical expression is available for the global minimum. Therefore, the minimization with respect to  $\mathbf{x}$  should be performed using a stochastic procedure. Among the many techniques for the stochastic optimization, we adopted simulated annealing [5]. However, different stochastic techniques can yield similar results.

Both the analytic minimization with respect to  $\mathbf{w}$  and the stochastic minimization with respect to  $\mathbf{x}$  are described in detail in [5]. Here, we simply recall that the analytic minimization needs the introduction of the following four assumptions regarding  $A_n$ : (1) all the transducer characteristics,  $A_n$ , are described by the same PDF  $f_A(A)$ ; (2)  $a_n$  and  $\gamma_n$  are independent random variables so that the joint PDF is separable,  $f_A(A) = f_a(a) f_\gamma(\gamma)$ , where  $f_a(a)$  is the PDF of the gain and  $f_\gamma(\gamma)$  is the PDF of the phase; (3)  $f_\gamma(\gamma)$  is an even function; and (4) the mean value of  $a_n$  is 1.

#### IV. RESULTS

In [5], an array aperture close to the wavelength at the lowest frequency is considered and the imperfections characterizing the microphones are supposed to be quite limited: standard deviation of 0.03 for the gain and 0.035 rad for the phase. If a square array of hydrophones with a side of 60 cm is designed to process frequencies equal to or higher than 1.5 kHz, the aperture turns out to be  $0.6\lambda$ ,  $\lambda$  being the wavelength at the lowest frequency. The utility of the superdirective performance is clear. Moreover, when low-cost hydrophones are addressed, the imperfections and fluctuations should be assumed much greater than those assumed for precision microphones. Consequently, we assume here that the standard deviations are 0.15 for the gain and 0.15 rad for the phase. Regarding the number of transducers, we assume the array to be composed of  $N = 36$  point-like omnidirectional hydrophones, and the frequency interval of interest from  $f_{min} = 1.5$  kHz to  $f_{max} = 12$  kHz (i.e., three octaves). At the highest frequency, an equispaced planar array of the same size requires 121 hydrophones over a  $11 \times 11$  grid to avoid grating lobes for whatever steering direction. The utility of the aperiodic layout is clear.

The filter-and-sum beamforming structure assumed FIR filters of the 20<sup>th</sup>-order (i.e.,  $L = 21$  taps). The region of the plane over which the BP is optimized is  $u \in [-1.6, 1.6]$ ,  $v \in [-1.6, 1.6]$ . This means that the steering can be performed inside a square pyramid whose apex is at the coordinate origin, the  $z$ -axis is the principal axis, and the angle between the two opposite triangular faces is  $75^\circ$ .

The optimized array layout obtained after  $10^4$  iterations of the combined minimization procedure is shown in Fig. 1.

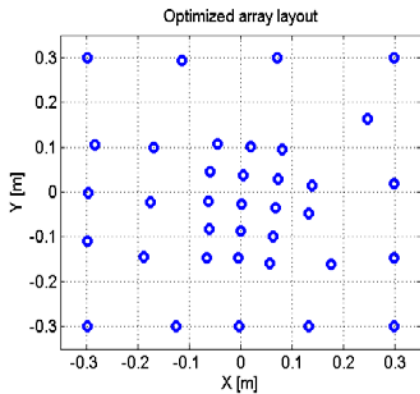


Fig. 1. Layout of the optimized array with 36 hydrophones.

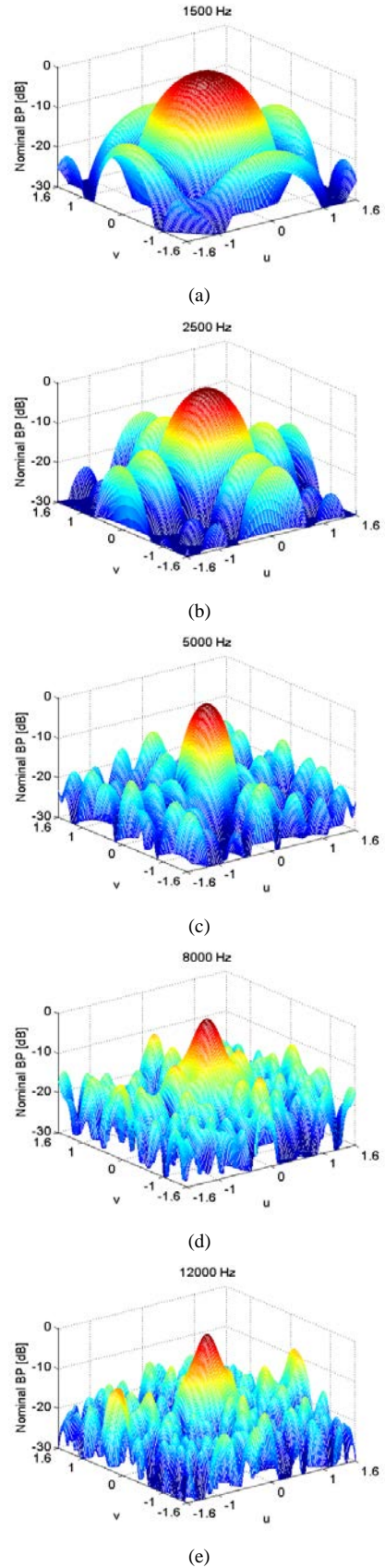


Fig. 2. Nominal BP for the optimized array and beamformer. The nominal BP is shown at (a) 1.5 kHz, (b) 2.5 kHz, (c) 5 kHz, (d) 8 kHz, and (e) 12 kHz.

The nominal BPs obtained using this array layout with the optimized FIR filters, for a set of frequency values spanning the range of interest (i.e., 1.5, 2.5, 5, 8, and 12 kHz), are shown in Fig. 2. As expected, the size of the main lobe decreases progressively as the frequency increases. To better assess this finding, the total width of the main lobe, measured at  $-3$  dB at an azimuth of  $0^\circ$  when the array is steered broadside, is displayed in Fig. 3(a). The main-lobe width decreases monotonically from approximately  $55^\circ$  at 1.5 kHz to approximately  $15^\circ$  at 12 kHz. The BP directivity is shown in Fig. 3(b) for broadside steering.

Finally, the white-noise gain (WNG) oscillations with frequency are shown in Fig. 3(c). The WNG is the improvement in SNR obtained using the array and considering spatially uncorrelated noise [6]. Because the array errors have an effect similar to that of a spatially uncorrelated noise, the WNG is considered a measure of the robustness of a superdirective array [4-7]. As a result of the large magnitude of the hydrophone mismatches, the value of the WNG is also positive at the lowest frequencies, which imposes a considerable robustness. The WNG level is closely related to the BP robustness through the expected floor level of the Mean Beam Power Pattern (MBPP). The latter is the mean value of the actual BP magnitude squared and which depends on the gain and phase PDFs of the hydrophones. [6]. The expected floor level is a specific level below which the actual BP is not expected to fall [6]. The MBPP for a frequency of 1.5 kHz was computed assuming the same error magnitude used in the optimization stage and is shown in Fig. 4. The expected floor level is approximately  $-14$  dB, close to the side-lobe level observed in the nominal BP. Therefore, no significant increase of the side-lobe level is expected as a consequence of the hydrophone errors.

## V. CONCLUSIONS

The use of the method proposed in [5] to design a sparse aperiodic array of hydrophones and the related filter-and-sum beamformer, in view of assembling a low-cost system for wideband passive imaging, has been assessed. At low frequency, the method in [5] provides robust superdirective performance by exploiting the PDFs of the array errors.

The adoption of only 36 low-cost hydrophones, placed over a square aperture with a side of 60 cm, allowed us to achieve a robust beam pattern with a main-lobe width ranging from  $55^\circ$  to  $15^\circ$  for a frequency interval from 1.5 kHz to 12 kHz. The side-lobe peak was of approximately  $-10$  dB for whatever steering direction inside a square pyramid with an angular aperture of  $75^\circ$ .

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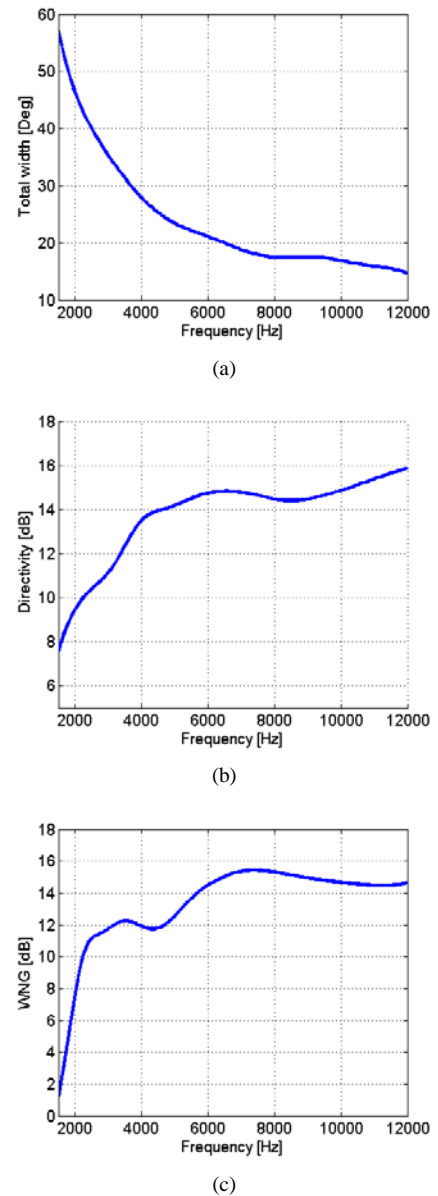


Fig. 3. (a) Main-lobe width at  $-3$  dB for broadside steering. (b) Directivity for broadside steering. (c) White noise gain.

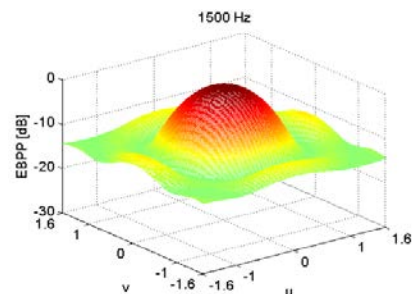


Fig. 4. Mean beam power pattern at 1.5 kHz for the optimized array and beamformer.

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